Problema: $AL0034$ **- Algebra - espressioni con i prodotti notevoli somma per differenza e quadrato di un binomio.**

Testo *[AL0034]* [1 2 1] Semplifica le seguenti espressioni utilizzando i prodotti notevoli di somma per differenza e quadrato di un binomio.

(a) $(1+3x)(1-3x)\left(1+9x^2\right)-(3+x)(3-x)\left(9+x^2\right)+80$

(b) $\left(x+\frac{2}{3}y\right)\left(\frac{2}{3}y-x\right)\left(\frac{4}{9}y^2-x^2\right)+\frac{32}{81}x^2y^2$

(c) $(4b-1)^2+(b-2)(b+2)\left(b^2-4\right)-\left(b^2+4\right)^2$

(d) $(3a-b)(3a+b)-(3a-b)^2$

(e) $(5ab-1)^2-25a^2b^2+10ab$

(f) $(m+n)(m-n)+\left(m^2-1\right)^2-m^4+n^2$

(g) $\left(2x^2-3y\right)\left(2x^2+3y\right)-\left(2x^2-3y\right)^2-18y^2$

(h) $\left(a^2+b^2\right)^2-4a^2b^2-\left(a^2-b^2\right)\left(a^2+b^2\right)$

(i) $\left(\frac{2}{3}x-3\right)^2+\frac{4}{9}(x+1)(x-1)-4x$

(j) $(x-1)(x+1)\left(x^2+1\right)-x^4+2$

(k) $\left(y^2-3\right)\left(y^2+3\right)+\left(y^2+1\right)^2-2y^2$

(l) $\left(\frac{1}{2}x+2\right)^2-\frac{1}{4}(x-2)(x+2)$

(m) $\left(2a^3-\frac{1}{4}b^2\right)\left(2a^3+\frac{1}{4}b^2\right)-\left(a^3-b^2\right)\left(a^3+b^2\right)+3a^6$

(n) $\left(9x^2+y^2\right)^2-(3x-y)^2(3x+y)^2$

(o) $(3x+2y)^2-(3x-2y)^2-24xy+1$

(p) $(x+2y)^2-(x-y)(x+y)-(x-2y)^2-3y(x+y)$

(q) $[(a-b)(a+b)]^2-\left(a^2-b^2\right)^2+\left(a^2+b^2\right)^2-a^4-b^4$

(r) $\left(3x-\frac{1}{2}y\right)^2-\left(\frac{1}{2}x+y\right)^2+3x(2-y)^2-3y^2\left(x-\frac{1}{4}\right)+4x(4y-3)$

(s) $(2a+b)^2(a-b)^2-2(3-b)^2(3+b)^2-\left(6b+2a^2\right)^2+a^2b[4a+3(b+8)]$

(t) $\xi=\left(x^3+x-x^2-1\right)\cdot(x+1)-\left(x^2+1\right)\left(x^2-1\right)$

(u) $\left(2x-\frac{1}{2}y\right)^2-\left(2x+\frac{1}{2}y\right)^2+4xy$

(v) $\left(x^2+2\right)\left(x^2-2\right)-\left(x^2-1\right)^2$

(w) $(x-1)(x+1)\left(x^2+1\right)\left(x^4+1\right)-x^8+5$

(x) $2(x+y)^2-(x+2y)(2x+y)$

(y) $\left(\frac{1}{3}a-3b\right)^2+2a\left(b-\frac{1}{18}a\right)$

(z) $(2x-3)^2-(2x+3)(2x-3)+12(x-1)$

Spiegazione In questo esercizio devi riconoscere ed usare i prodotti notevoli ed utilizzarli.

Svolgimento Qui di seguito le soluzioni degli esercizi proposti

(a)

$$\left(1-9x^2\right)\left(1+9x^2\right)-\left(9-x^2\right)\left(9+x^2\right)+80$$
$$\left(1-81x^4\right)-\left(81-x^4\right)+80$$
$$-80-80x^4+80$$
$$-80x^4$$

.

(b)

$$\left(\frac{4}{9}y^2-x^2\right)\left(\frac{4}{9}y^2-x^2\right)+\frac{32}{81}x^2y^2$$
$$\left(\frac{16}{81}y^4-\frac{32}{81}x^2y^2+x^4\right)+\frac{32}{81}x^2y^2$$
$$\frac{16}{81}y^4+x^4$$

.

(c)

$$(4b-1)^2+(b-2)(b+2)\left(b^2-4\right)-\left(b^2+4\right)^2$$
$$16b^2-8b+1+\left(b^2-4\right)\left(b^2-4\right)-b^4-16-8b^2$$
$$16b^2-8b+1-\left(b^4+16-8b^2\right)-b^4-16-8b^2$$
$$16b^2-8b+1-b^4-16+8b^2-b^4-16-8b^2$$
$$-8b+1$$

.

(d) $6ab-2b^2$.

(e) 0.

(f) 1.

(g) $12x^2y-27y^2$.

(h) $2b^4$.

(i) $\frac{17}{3}$.

(j) 1.

(k) $2y^4-8$.

(l) $2x+5$.

(m) $4a^6+\frac{15}{16}b^4$.

(n) $36x^2y^2$.

(o) 1.

(p) $-x^2+5xy-2y^2$.

(q) $2a^2b^2$.

(r) $\frac{35}{4}x^2$.

(s) $2ab^3-b^4-162$.

(t) $\xi=\left(x^3+x-x^2-1\right)\cdot(x+1)-\left(x^2+1\right)\left(x^2-1\right)$
$\xi=x^4-x^3+x^2-x+x^3-x^2+x-1-\left(x^4-1\right)=0$
oppure in alternativa
$\xi=\left(x^3+x-x^2-1\right)\cdot(x+1)-\left(x^2+1\right)\left(x^2-1\right)$
$\xi=\left[x\left(x^2+1\right)-\left(x^2+1\right)\right]\cdot(x+1)-\left(x^2+1\right)\left(x^2-1\right)$
$\xi=\left(x^2+1\right)(x-1)\cdot(x+1)-\left(x^2+1\right)\left(x^2-1\right)$
$\xi=\left(x^2+1\right)\left(x^2-1\right)-\left(x^2+1\right)\left(x^2-1\right)=0$.

(u) $4x^2-2xy+\frac{1}{4}y^2-\left(4x^2+2xy+\frac{1}{4}y^2\right)+4xy$
$4x^2-2xy+\frac{1}{4}y^2-4x^2-2xy-\frac{1}{4}y^2+4xy=0.$

(v) $x^4-4-\left(x^4-2x^2+1\right)$
$x^4-4-x^4+2x^2-1=2x^2-5.$

(w) $\left(x^2-1\right)\left(x^2+1\right)\left(x^4+1\right)-x^8+5$
$\left(x^4-1\right)\left(x^4+1\right)-x^8+5$
$x^8-1-x^8+5=4.$

(x) $2\left(x^2+2xy+y^2\right)-\left(2x^2+xy+4xy+2y^2\right)$
$2x^2+4xy+2y^2-2x^2-5xy-2y^2=-xy.$

(y) $\frac{1}{9}a^2-2ab+9b^2+2ab-\frac{1}{9}a^2=9b^2.$

(z) $4x^2-12x+9-\left(4x^2-9\right)+12x-12$
$4x^2-12x+9-4x^2+9+12x-12=6.$