

Problema: AL0036 - Algebra letterale

Testo [AL0036] [2★ 2🕒 1📖] Semplifica le seguenti espressioni.

(a)
$$\sqrt{\frac{x+y}{x-y} - \frac{x-y}{x+y}} \cdot \sqrt{\frac{1}{1 - \frac{y-x}{x+y}}} \cdot \sqrt[6]{\frac{(x-y)^2}{3x}}$$

(b)
$$\frac{x\sqrt{y}}{x+y} \sqrt{\frac{y^2+xy}{x^2}} - \sqrt{\frac{x^2}{x+y}} \cdot \frac{2\sqrt{x^2y+y^3+2xy^2}}{\sqrt{4x^2y}}$$

(c)
$$\sqrt[6]{\frac{x^2}{x+y}} \cdot \sqrt[6]{\frac{(x+y)^3}{y^2}} \cdot \sqrt[3]{\frac{y}{x(x+y)}} \cdot \sqrt[4]{\frac{y(x+y)}{x}} \cdot \sqrt[4]{\frac{x}{y}}$$

Spiegazione In questo esercizio devi riconoscere ed usare i prodotti notevoli ed utilizzare le regole dei radicali.

Svolgimento Qui di seguito le soluzioni degli esercizi proposti

(a)

$$\begin{aligned} & \sqrt{\frac{x+y}{x-y} - \frac{x-y}{x+y}} \cdot \sqrt{\frac{1}{1 - \frac{y-x}{x+y}}} \cdot \sqrt[6]{\frac{(x-y)^2}{3x}} \\ & \sqrt{\frac{(x+y)^2 - (x-y)^2}{(x-y)(x+y)}} \cdot \sqrt{\frac{x+y}{x+y-y+x}} \cdot \sqrt[6]{\frac{(x-y)^2}{3x}} \\ & \sqrt{\frac{(x+y+x-y)(x+y-x+y)}{(x-y)(x+y)}} \cdot \sqrt{\frac{x+y}{2x}} \cdot \sqrt[6]{\frac{(x-y)^2}{3x}} \\ & \sqrt{\frac{4xy}{(x-y)(x+y)}} \cdot \sqrt{\frac{x+y}{2x}} \cdot \sqrt[6]{\frac{(x-y)^2}{3x}} \\ & \sqrt{\frac{4xy}{(x-y)(x+y)}} \cdot \frac{x+y}{2x} \cdot \sqrt[6]{\frac{(x-y)^2}{3x}} \\ & \sqrt{\frac{2y}{(x-y)}} \cdot \sqrt[6]{\frac{(x-y)^2}{3x}} \\ & \sqrt[6]{\frac{8y^3}{(x-y)^3}} \cdot \sqrt[6]{\frac{(x-y)^2}{3x}} \\ & \sqrt[6]{\frac{8y^3}{3x(x-y)}} \end{aligned}$$

(b)

$$\begin{aligned} & \sqrt{x^2y} \frac{y(y+x)}{x^2(x+y)^2} - \sqrt{\frac{x^2}{x+y}} \cdot \frac{2\sqrt{y(x+y)^2}}{2\sqrt{x^2y}} \\ & \sqrt{\frac{y^2}{(x+y)}} - \sqrt{\frac{x^2}{x+y}} \cdot \frac{\sqrt{y(x+y)^2}}{\sqrt{x^2y}} \\ & \sqrt{\frac{y^2}{(x+y)}} - \sqrt{\frac{x^2}{x+y} \frac{y(x+y)^2}{x^2y}} \\ & \sqrt{\frac{y^2}{(x+y)}} - \sqrt{x+y} \\ & \sqrt{\frac{y^2(x+y)}{(x+y)^2}} - \sqrt{x+y} \\ & \sqrt{x+y} \left(\frac{y}{x+y} - 1 \right) \\ & - \frac{x}{x+y} \sqrt{x+y} \\ & - \frac{x}{\sqrt{x+y}} \end{aligned}$$

(c)

$$\begin{aligned} & \sqrt[6]{\frac{x^2}{x+y}} \cdot \sqrt[6]{\frac{(x+y)^3}{y^2}} \cdot \sqrt[3]{\frac{y}{x(x+y)}} \cdot \sqrt[4]{(x+y)} \\ & \sqrt[6]{\frac{x^2}{x+y}} \cdot \frac{(x+y)^3}{y^2} \cdot \frac{y^2}{x^2(x+y)^2} \cdot \sqrt[4]{(x+y)} \\ & \sqrt[4]{(x+y)} \end{aligned}$$