

Problema: *AL0037 - Algebra letterale*

Testo [\[AL0037\]](#) [1½★ 2🕒 2📖] Semplifica le seguenti espressioni.

(a)
$$\frac{(x^2 - y^2)^{\frac{1}{2}}}{(x^{-1} - y^{-1})} \cdot \frac{(x - y)^{\frac{1}{2}}}{y^{-2} - x^{-2}} \cdot \frac{(xy)^{-3}}{(x + y)^{\frac{1}{2}}}$$

(b)
$$\frac{(x + y)^{\frac{1}{2}}}{(x + y)^{\frac{1}{2}} + (x - y)^{\frac{1}{2}}} + \frac{(x + y)^{\frac{1}{2}}}{(x + y)^{\frac{1}{2}} - (x - y)^{\frac{1}{2}}}$$

(c)
$$a^{\frac{3}{2}} \sqrt[4]{\frac{(a^4 + b^4)^3}{2^4 a^3}} \cdot \sqrt[3]{\frac{a^4 + b^4}{a^4 + a^3 b}} \cdot \sqrt[12]{\frac{a^4 + a^3 b}{a^4 + b^2}} \quad \text{con } a > 0$$

(d)
$$\sqrt{\frac{16x^3 + 16x^2}{y^3 - y^2}} + \frac{1}{y} \sqrt{\frac{(x^3 + x^2)(y + 1)}{y^2 - 1}} - x \sqrt{\frac{4x + 4}{y^3 - y^2}} \quad \text{con } x \geq 0; y > 1$$

Spiegazione

Svolgimento Qui di seguito le soluzioni degli esercizi proposti

(a)

$$\frac{(x^2 - y^2)^{\frac{1}{2}}}{(x^{-1} - y^{-1})} \cdot \frac{(x - y)^{\frac{1}{2}}}{y^{-2} - x^{-2}} \cdot \frac{(xy)^{-3}}{(x + y)^{\frac{1}{2}}}$$

$$\frac{\sqrt{(x - y)(x + y)}}{\left(\frac{1}{x} - \frac{1}{y}\right)} \cdot \frac{\sqrt{(x - y)}}{\frac{1}{y^2} - \frac{1}{x^2}} \cdot \frac{1}{x^3 y^3 \sqrt{(x + y)}}$$

$$\frac{xy \sqrt{(x - y)(x + y)}}{y - x} \cdot \frac{x^2 y^2 \sqrt{(x - y)}}{x^2 - y^2} \cdot \frac{1}{x^3 y^3 \sqrt{(x + y)}}$$

Semplificando tutti i termini

$$\frac{1}{x^2 - y^2}$$

(b)

$$\sqrt{(x + y)} \left(\frac{1}{\sqrt{(x + y)} + \sqrt{(x - y)}} + \frac{1}{\sqrt{(x + y)} - \sqrt{(x - y)}} \right)$$

$$\sqrt{(x + y)} \left(\frac{\sqrt{(x + y)} - \sqrt{(x - y)} + \sqrt{(x + y)} + \sqrt{(x - y)}}{x + y + x - y} \right)$$

$$\sqrt{(x + y)} \left(\frac{2\sqrt{(x + y)}}{2x} \right)$$

$$\frac{x + y}{x}$$

(c)

$$a^{\frac{3}{2}} \sqrt[4]{\frac{(a^4 + b^4)^3}{2^4 a^3}} \cdot \sqrt[3]{\frac{a^4 + b^4}{a^4 + a^3 b}} \cdot \sqrt[12]{\frac{a^4 + a^3 b}{a^4 + b^2}}$$

$$\frac{1}{2} a^{\frac{3}{2}} \sqrt[12]{\frac{(a^4 + b^4)^9}{a^9}} \cdot \sqrt[3]{\frac{a^4 + b^4}{a^3 (a + b)}} \cdot \sqrt[12]{\frac{a^3 (a + b)}{a^4 + b^4}}$$

$$\frac{1}{2} \sqrt{a} \sqrt[12]{\frac{(a^4 + b^4)^9}{a^9}} \cdot \sqrt[3]{\frac{a^4 + b^4}{(a + b)}} \cdot \sqrt[12]{\frac{a^3 (a + b)}{a^4 + b^4}}$$

$$\frac{1}{2} \sqrt{a} \sqrt[12]{\frac{(a^4 + b^4)^9}{a^9}} \cdot \sqrt[12]{\frac{(a^4 + b^4)^4}{(a + b)^4}} \cdot \sqrt[12]{\frac{a^3 (a + b)}{a^4 + b^4}}$$

$$\frac{1}{2} \sqrt{a} \sqrt[12]{\frac{(a^4 + b^4)^9}{a^9} \cdot \frac{(a^4 + b^4)^4}{(a + b)^4} \cdot \frac{a^3 (a + b)}{a^4 + b^4}}$$

$$\frac{1}{2} \sqrt{a} (a^4 + b^4) \sqrt[12]{\frac{1}{a^9} \cdot \frac{1}{(a + b)^4} \cdot a^3 (a + b)}$$

$$\frac{1}{2} (a^4 + b^4) \sqrt[12]{\frac{1}{(a + b)^3}}$$

$$\frac{1}{2} (a^4 + b^4) \sqrt[4]{\frac{1}{a + b}}$$

(d)

$$\frac{4x}{y} \sqrt{\frac{x + 1}{y - 1}} + \frac{x}{y} \sqrt{\frac{(x + 1)(y + 1)}{(y - 1)(y + 1)}} - \frac{2x}{y} \sqrt{\frac{x + 1}{y - 1}}$$

$$\frac{4x}{y} \sqrt{\frac{x + 1}{y - 1}} + \frac{x}{y} \sqrt{\frac{x + 1}{y - 1}} - \frac{2x}{y} \sqrt{\frac{x + 1}{y - 1}}$$

$$\frac{3x}{y} \sqrt{\frac{x + 1}{y - 1}}$$

3.4 polinomi - fattorizzazione